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Sixth graders' reasoning ability on invalid conditional inferences predicts concurrent algebraic achievement

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$$\frac{3x}{4} = 5 + 2x$$

Highlight

- Established relation between conditional reasoning ability and algebraic achievement
- Found underlying structure of conditional reasoning ability among sixth graders
- Only the ability to identify invalid conditional inferences (i.e., affirmation of consequent (AC) and denial of antecedent (DA)) in true and false contexts significantly predicted concurrent algebraic achievement
- Ability to detect violation of arithmetic principles partially mediated the relation

Introduction

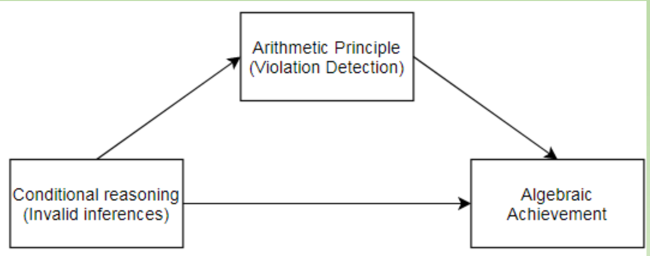
Logical reasoning has been thought to be associated with mathematics abilities (Morsanyi & Szücs, 2014; Nunes, Bryant, Barros, & Sylva, 2012). However, little empirical evidence has demonstrated a clear linkage (cf., Wong, 2018).

Conditional reasoning involves inferences based on truth value of antecedent or consequent of a premise. Four forms of inferences can be made, namely modus ponens (MP), modus tollens (MT), AC, and DA. Children's reasoning ability differs on these inferences and the differences also vary in terms of the truth value of the premise context (Markovits, 2000). However, it is still unknown if these variations imply different cognitions. The underlying factor structure is yet to be established.

Conditional reasoning is thought to be related to **algebra**, given that both stress on the comprehension of "If, then" statements. Also, during the reasoning process from the premise and translation of word problems, formation of mental representation about the relations between objects is needed.

Understanding of **arithmetic principles** (regularities behind operations) may mediate this relation, in particular the detection of principle violation. To reason correctly for **AC and DA** inferences, it involves the discovery of counterexamples (Morsanyi, McCormack, & O'Mahony, 2018). Similarly, acquisition of arithmetic principles is facilitated with the detection of deviation from the principles.

Proposed model



Methodology

Participants: Sixth graders in mainstream primary school (n = 101; Mean age = 12.17 years old)

Procedures: As part of a larger longitudinal study, individual assessment was administered to measure their conditional reasoning ability, algebraic achievement, arithmetic principle understanding, and a number of other cognitive abilities

Measures:

- Conditional reasoning (24 items)

(TRUE PREMISE) Every HKU student uses Zoom for online lectures.		
Truth value of antecedent/consequent	Type of Inference	Validity
If Anna is not a HKU student, does she use Zoom for online lectures?	Denial of Antecedent (DA)	Invalid
(FALSE PREMISE) If water is put into the freezer, it boils.		
Truth value of antecedent/consequent	Type of Inference	Validity
If water boils, is the water put into freezer?	Affirmation of Consequent (AC)	Invalid

- Detection of violation of arithmetic principles (48 items)

Determine whether the following statements are correct or not.	
A + B – B = A (Inversion)	✓ / ×

- Algebraic word problem solving (10 items)
- Control variables
  - Domain-general cognitive abilities
    - Raven's progressive matrices
    - Working memory
  - Domain-specific cognitive abilities
    - Reading comprehension
    - Fraction and decimal magnitude representation
  - Past mathematics achievement

Results

1) Exploratory factor analysis: 4-factor structure (factor loadings in parentheses)

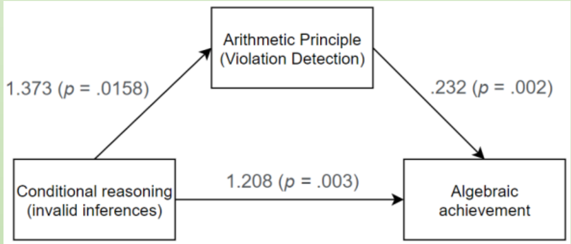
Factor 1	Factor 2	Factor 3	Factor 4
TrueAC (.792)	AbstractMP (-.603)	TrueMT (.710)	TrueMP (.855)
TrueDA (.803)	AbstractMT (-.682)	FalseMP (.788)	
FalseAC (.790)	AbstractAC (.704)	FalseMT (.736)	
FalseDA (.755)	AbstractDA (.649)		

Results (Cont'd)

2) Hierarchical linear regression: Factor 1 (invalid conditional inferences in true and false contexts) significantly predicted algebraic word problem solving performance

Step	Predictor	ΔR^2	ß	Sig.
1	Control variables	.240	N/A	<.001
2	Factor 1	.117	.369	<.001
	Factor 2		-.047	.643
	Factor 3		-.122	.245
	Factor 4		.009	.925

3) Mediation analysis: Partial mediation by arithmetic principle understanding (Effect size of indirect effect of conditional reasoning on algebraic achievement = .318, 95% BCaCI [.0622, .6799]



Discussion

- Discovered the factor structure of conditional reasoning ability among children in upper elementary school level
- Investigation of the relative importance of conditional inferences on a specific mathematical domain (i.e., algebra), with the effects of other known predictors adjusted (in response to Wong (2018))
- Proposed a concrete mechanism behind logic-math relation
  - Potential intervention on domain-general cognitive ability (conditional reasoning) for low achievers in algebra
- Call for attention on the significance of arithmetic principles to mathematics learning
  - Incorporation into mathematics curriculum

Reference

Markovits, H. (2000). A mental model analysis of young children's conditional reasoning with meaningful premises. *Thinking & Reasoning*, *6*(4), 335-347.

Morsanyi, K., McCormack, T., & O'Mahony, E. (2018). The link between deductive reasoning and mathematics. *Thinking & Reasoning*, *24*(2), 234-257.

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